INVESTIGATION ON DIAMAGNETIC LEVITATION OF GRAPHITE SHEETS

Members:
Jethro Chai Tsin Kai (Hwa Chong Institution)
Kevin Goh Ke Ming
(NUS High School of Mathematics and Science)
Soh En Qi Chloe (Raffles Institution)

Mentor:
Wee Wei Hsiung (DSO National Laboratories)

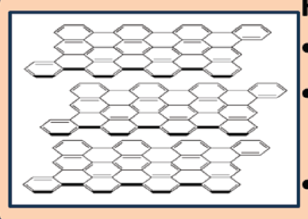
Introduction

Diamagnetic materials (χ <0) [1] can **levitate stably and passively** in a magnetic field
Graphite can serve as alternative to superconductor/electromagnet-based levitation

Aim: Model and Experimentally investigate behaviour of graphite sheet as a basis for designing better sensors and devices making use of diamagnetic levitators

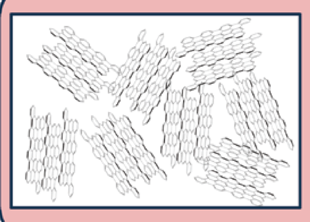
Objectives:

- Investigate behaviour and oscillations of isostatic graphite alongside pyrolytic graphite
- Verify empirically trends in with changes in temperature, force and dimensions



Pyrolytic

- Aligned layers of graphene
- Unpredictable flaking along layers
- Anisotropic susceptibility



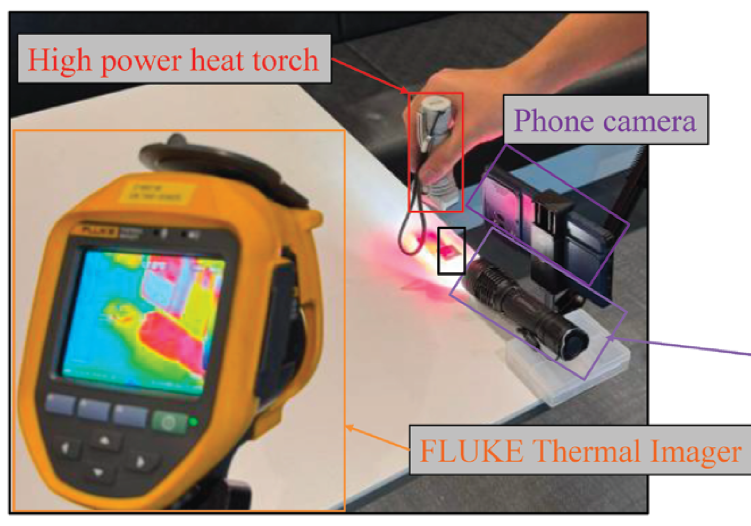
Isostatic

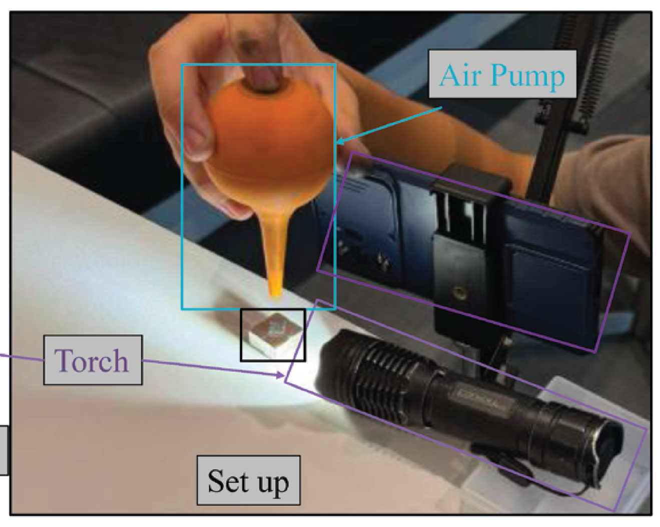
- Varying alignment
- Mechanically stable, does not flake off
- Isotropic susceptibility

Experimental Methods

Measuring levitation height change with temperature

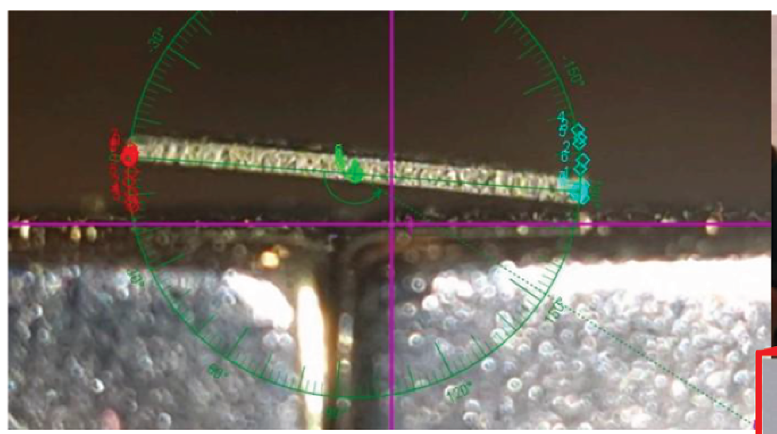
Tracking oscillation after disturbance
Measurement repeated with top view recorded

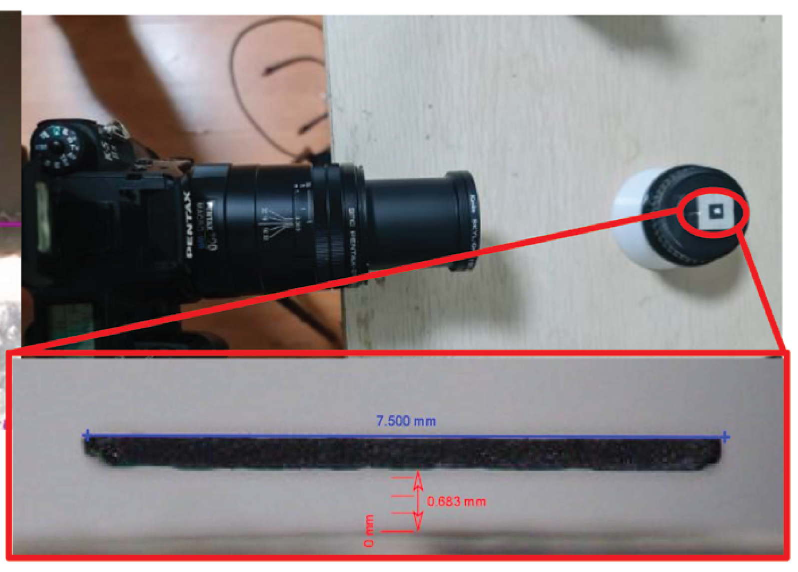




Tracking tilting angle displacement in Tracker App

Obtaining static levitation height of sheet carrying varying masses
Levitation height measured in Tracker App





Theoretical Methods

As commonly known,

$$\Phi_m = \frac{\nabla}{4\pi} \cdot \iiint \frac{\boldsymbol{M}(\boldsymbol{r}'')}{|\boldsymbol{r}-\boldsymbol{r}''|} d^3\boldsymbol{r}'' \quad (1)$$

Poisson's magnetic scalar potential

Include boundary conditions →

r'' is a point on the magnet

$$\Phi_m(\boldsymbol{r}) = \frac{M_0}{\pi} \int_{-\frac{W_x}{2}}^{\frac{W_x}{2}} \int_{-\frac{W_y}{2}}^{\frac{W_y}{2}} \frac{1}{|\boldsymbol{r}-\boldsymbol{r}''|} dy''\, dx'' \Big|_{z''=-W_z}^{0} \quad (2)$$

Poisson's magnetic scalar potential

$$\boldsymbol{B}(\boldsymbol{r}) = -\nabla\Phi_m(\boldsymbol{r}) \quad (3)$$

Magnetic Field

$$u_m(\boldsymbol{s},\boldsymbol{\theta}) = \frac{1}{2\mu_0}\left(\chi_x B'^2_x + \chi_y B'^2_y + \chi_z B'^2_z\right) \quad (4)$$

Magnetic Potential Energy Density

$$R[\boldsymbol{\theta}] = R_x(\theta_x) R_y(\theta_y) R_z(\theta_z)$$
$$\boldsymbol{B}' = R[\boldsymbol{\theta}]^{-1} \boldsymbol{B}(R[\boldsymbol{\theta}] \cdot \boldsymbol{r}' + \boldsymbol{s}) \quad (5)$$

Davenport chain rotation

$$U_m(\boldsymbol{s},\boldsymbol{\theta}) = \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}} \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} \int_{0}^{L_z} u_m(R[\boldsymbol{\theta}] \cdot \boldsymbol{r}' + \boldsymbol{s}, \boldsymbol{\theta})\, d^3\boldsymbol{r}' \quad (6)$$

Magnetic Potential Energy

$$U(\boldsymbol{s},\boldsymbol{\theta}) = U_m(\boldsymbol{s},\boldsymbol{\theta}) + m\, g\, s_z \quad (7)$$

Total Potential Energy

$$\boldsymbol{F} = -\nabla U(\boldsymbol{s}_0,\boldsymbol{\theta}_0) = 0 \quad (8)$$

Equilibrium position

$\boldsymbol{s}_0, \boldsymbol{\theta}_0$ →

$$K = \nabla\nabla^T U(\boldsymbol{s}_0,\boldsymbol{\theta}_0)$$

Hessian (Stiffness)

$$H = G^{-\frac{1}{2}} \cdot K \cdot G^{-\frac{1}{2}}$$

Normalised Hessian

$$f_a = \frac{\sqrt{\text{Eigenvalues}[H]}}{2\pi}$$

Modal Frequencies

$$\boldsymbol{V}_a = \text{Eigenvectors}[H]$$

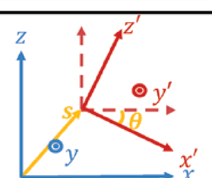
Modes

$$G = \text{Diag}[m, m, m, I_x, I_y, I_z]$$

Mass matrix

$$I_z = \frac{1}{12\pi} m(L_x^2 + L_y^2)$$

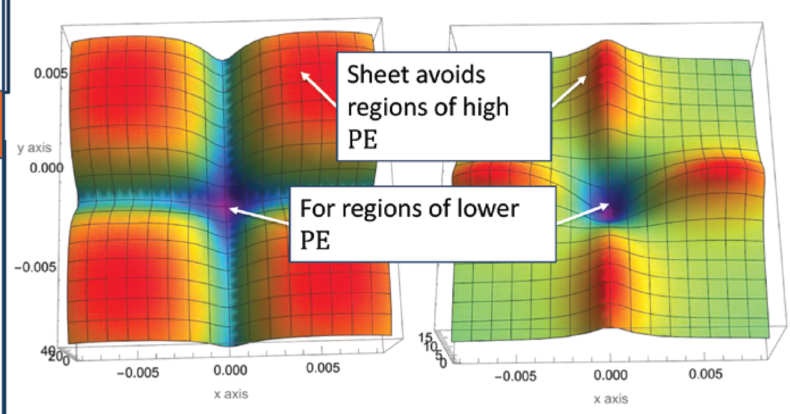
Moment of Inertia



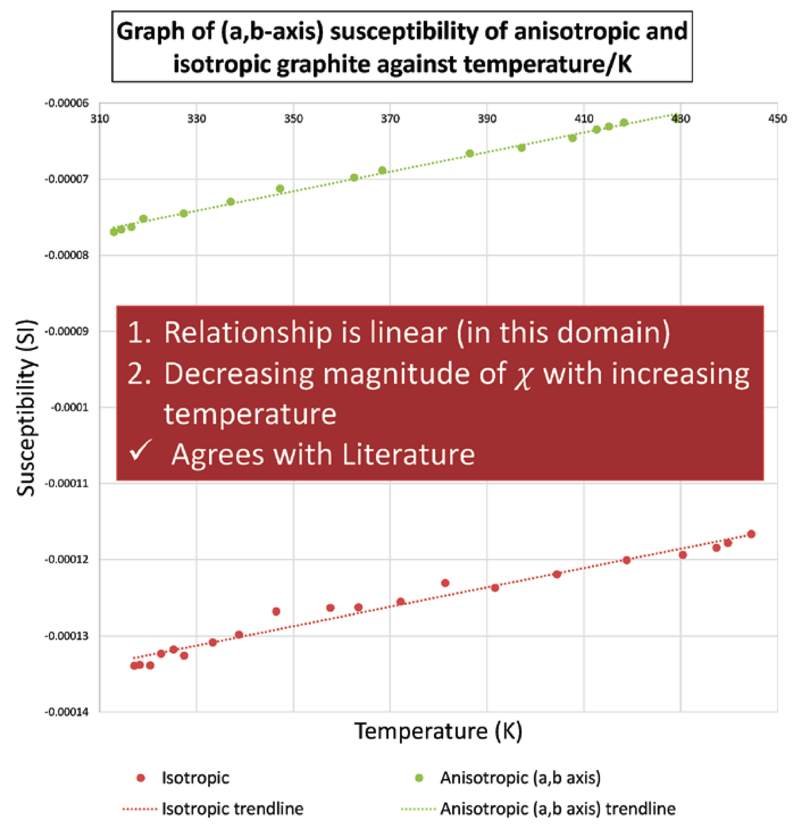
Results & Discussion

Anisotropy & Isotropy on Equilibrium Position

Magnetic Potential Energy Density in x-y axis



Effect of Temperature on Diamagnetic Property

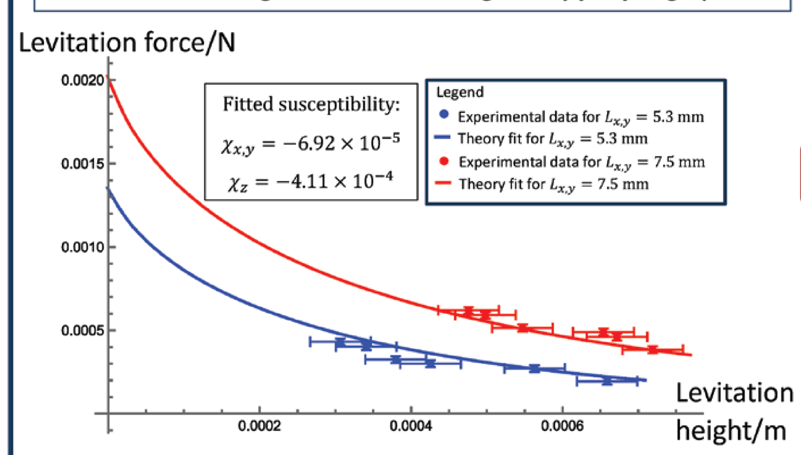


1. Relationship is linear (in this domain)
2. Decreasing magnitude of χ with increasing temperature

✓ Agrees with Literature

- As temperature rises, electrons transition from the valence band to the conduction band [3].
- electrons in the conduction band contribute less to diamagnetism

Levitation Force at Different Levitation Heights

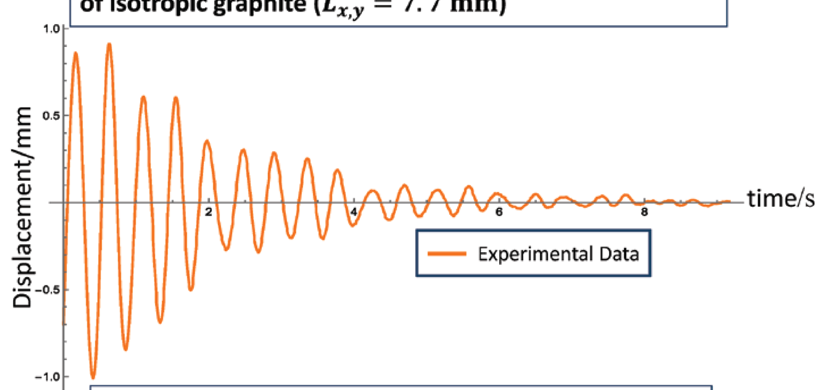


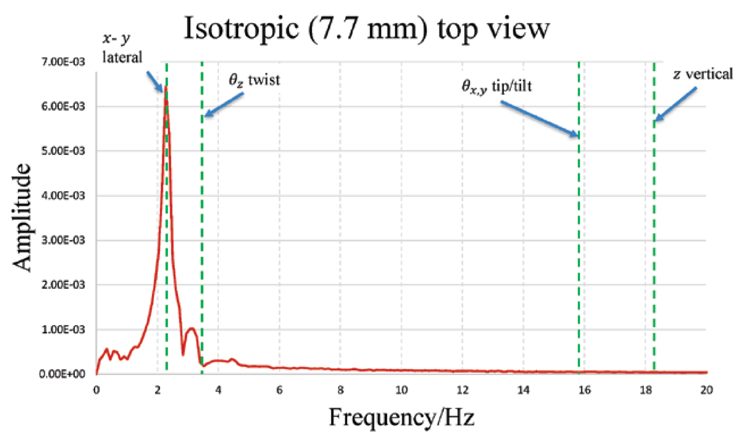
- Levitation force is **greater** when **closer** to the magnet.
- **Under-prediction** of $L_{x,y} = 7.5$mm graphite sheet
  - Small differences in thickness between the sheets

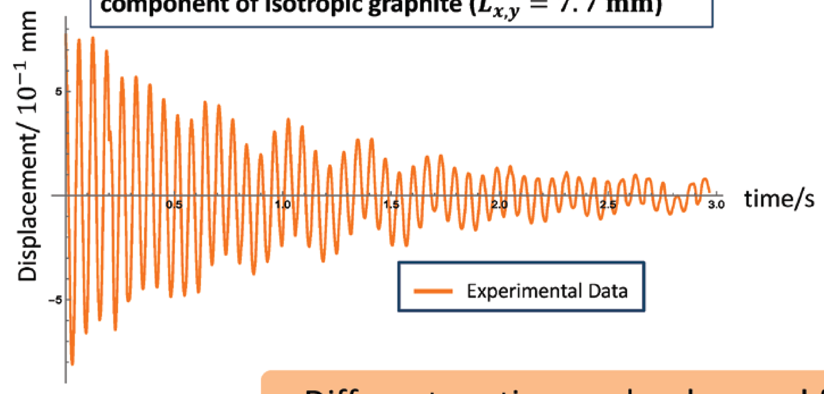
Force increases with decreasing levitation height

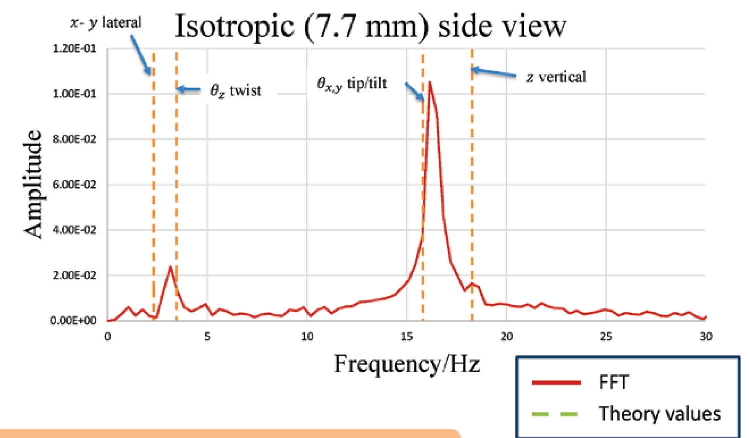
Force increases with increasing sheet size

Oscillation Modal Frequency



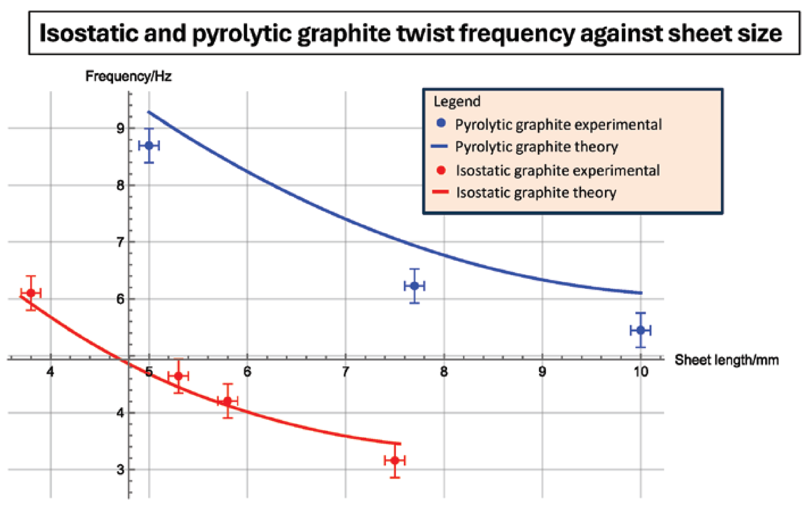






Different motion peaks observed from top view and side view

Size of Graphite Sheet and Oscillation Frequency



- **Increasing** sheet size **decreased** oscillation frequency across all modes
- **Pyrolytic** graphite has **higher** twisting frequency
  - Correlates with theoretical prediction
- **Pyrolytic** graphite **frequencies deviate more** from theory

✓ Overall **close** agreement with theory
➢ **verified accuracy of model**

Conclusion

- **Difference in equilibrium positions result of anisotropy**
- **Increasing sheet size** → Decreasing rate of **decrease in frequency** (agrees with theory)
- Temperature increase → **decreased** diamagnetic susceptibility magnitude
- Characterisation paves the way for use in microfluids, geological survey and other industries

Acknowledgements

We would like to extend our gratitude to the following individuals:
Dr Xianfeng Chen (A*STAR), for providing some pyrolytic graphite sheets for our experiment.
Dr Wee Wei Hsiung (DSO/ES), for being our mentor and supporting us along the course of this project.
Mr Leslie Lam (DSO/ES), and other members of the Emerging Division who also helped out.
Ms Emeline Sim (DSO) for her kind assistance with the administrative aspect of the project.

References

[1] Earnshaw, S. (1848). On the Nature of the Molecular Forces which Regulate the Constitution of the Luminiferous Ether. On The Nature of the Molecular Forces Which Regulate the Constitution of the Luminiferous Ether, 7, 97. https://ui.adsabs.harvard.edu/abs/1848TCaPS. . .7. . .97E/abstract
[2] Tian, S., Jadeja, K., Kim, D., Hodges, A., Hermosa, G. C., Cusicanqui, C., Lecamwasam, R., Downes, J. E., & Twamley, J. (2024). Feedback cooling of an insulating high-Q diamagnetically levitated plate. Applied Physics Letters, 124(12). https://doi.org/10.1063/5.0189219
[3] Banerjee, D., & Bhattacharya, R. (1990). Temperature variation of diamagnetic susceptibility of graphite. In Indian Association for the Cultivation of Science, Indian J. Phys. (Vol. 64, Issue 4, pp. 316–320). https://core.ac.uk/download/pdf/156953823.pdf
[4] Gomari, Sepideh & Namin, Parvin & Ghasemi, Ismaeil. (2019). Polymer-Graphene Nanoplatelets Nanocomposites: Properties and Applications. Iranian journal of polymer science & technology. 32. 101-121. 10.22063/JIPST.2019.1647.